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Quasi-localization and quasi-mobility edge for light atoms mixed with heavy ones

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Abstract. A mixture of light and heavy atoms is considered. We study the kinetics of the light atoms, scattered by the heavy ones, the latter undergoing slow diffusive motion. In three-dimensional space we claim the existence of a crossover region (in energy), which separates the states of the light atoms with fast diffusion and the states with slow diffusion; the latter is determined by the dephasing time. For the two dimensional case we have a transition between weak localization, observed when the dephasing length is less than the localization length (calculated for static scatterers), and strong localization observed in the opposite case.

PACS. 72.15.Rn Localization effects (Anderson or weak localization)

Mixtures of different species of cold atoms present an interesting field of many particle physics. Two or more different types of atoms can be mixed, where one type of atoms can be relatively light (e.g. ^{6}Li), and the other type is heavy (e.g. ^{87}Rb). Quantum tunneling of light atoms is a phenomenon, interesting both from an experimental and theoretical point of view. The heavy atoms serve as slow moving scatterers for the light atoms. Lately, it was realized that ultracold atomic gases appear very convenient for experimental studies of Anderson localization of the light atoms, both for the case of Bose-Einstein condensates, and for fermionic gases [1–11].

Kinetics of classical particles in a disordered medium can be described by the Boltzmann equation. The most drastic manifestation of the difference between the kinetics of classical particles and that of quantum ones is Anderson localization. It is well known that for $d = 1$ and $d = 2$, where d is the dimensionality of space, all the states are localized, and for $d = 3$ there exists a mobility edge E_c, the energy which separates the states with finite diffusion coefficient and states with the diffusion coefficient being exactly equal to zero (For reviews, see e.g. Refs. [12–14]). All this is true provided the disorder is static. A natural question arises: what happens with this picture when the scatterers slowly move?

To answer this question we need some quantitative theory of localization. As such we will use the self-consistent localization theory by Vollhard and Wölfle [15]. Of crucial importance in the above mentioned theory are maximally crossed diagrams (the sum of all such diagrams is called the Cooperon) for the two-particle Green function. The calculations of these diagrams for the case of moving scatterers were done in the paper by Golubentsev [16].

One should notice that we consider the heavy atoms as classical objects whose diffusive motion is not affected by localization effects. On the other hand, we consider the light atoms as quantum objects. Thus the temperature of the atom gases should satisfy the inequalities [17]

$$\frac{\hbar^2}{M}N^{2/d} \ll T \ll \frac{\hbar^2}{m}n^{2/d}, \tag{1}$$

where M and N are the mass and concentration of heavy atoms respectively, m and n are the mass and concentration of light atoms and T is the temperature. The large ratio between the masses of the two types of atoms considered is crucial for the applicability of the methods used in this work also because following reference [16], we shall ignore the change of the energy of the light atoms as a result of a scattering by a heavy one.

In the first part of the present paper we reproduce the results by Golubentsev (trivially generalized for the arbitrary dimensionality of space). In the second part we use the results for the Cooperon as an input for the self-consistent localization theory, which we modify to take into account the slow motion of scatterers. In the third part we discus the results obtained.

The quantum particles are scattered by the potential

$$V(r,t) = V\sum_a \delta\left(r - r_a(t)\right). \tag{2}$$

Define the correlator

$$K(r-r',t-t') = \langle V(r,t)V(r',t')\rangle. \tag{3}$$

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To leading order in the density of scatterers we have for the Fourier component of the correlator

$$K(q,t)=V^2\left\langle \int \exp\{(i\mathbf{q}(\mathbf{r}-\mathbf{r}')\} \times dr dr' \sum_a \delta(\mathbf{r}-\mathbf{r}_a(t)) \sum_{a'} \delta(\mathbf{r}'-\mathbf{r}'_{a'}(0)) \right\rangle = V^2 \sum_a \langle \exp\{iq(\mathbf{r}_a(t)-\mathbf{r}_a(0))\}\rangle = nV^2 f(\mathbf{q},t), \quad (4)$$

where n is the scatterer density. We consider the case when the scatterers undergo slow diffusive motion. In the ballistic case

$$f(\mathbf{q},t)=\exp\left(-\frac{\mathbf{q}^2T}{2M}t^2\right), \qquad |t|\ll\tau_{imp}, \quad (5)$$

In the diffusive case

$$f(\mathbf{q},t)=\exp\left(-\frac{\mathbf{q}^2T\tau_{imp}}{2M}|t|\right), \qquad |t|\gg\tau_{imp}, \quad (6)$$

where we have used the fact that

$$\langle \mathbf{v}_{imp}^2\rangle=\frac{dT}{M}, \quad (7)$$

and τ_{imp} is the mean free time of the scatterers.

For the Cooperon we get [16]

$$C_E(\mathbf{q})=\int_0^\infty \exp\left\{-D(E)q^2t-\frac{1}{\tau}\int_0^t(1-f_{t'})dt'\right\}dt, \quad (8)$$

where E is the energy of each of the two quantum particle lines in the Cooperon diagram, and q is the sum of their momenta (see Fig. 1). Also

$$\frac{1}{\tau}=\begin{cases} nV^2k^2/\pi v & d=3\\ nV^2k/v & d=2\\ nV^2/v & d=1.\end{cases} \quad (9)$$

We'll assume that $\tau\ll\tau_{imp}$. The quantity f_t is $f(\mathbf{k})$ averaged with respect to the iso-energetic surface. We obtain

$$f_t=\begin{cases} y_d\left(\frac{t^2}{\tau_\lambda^2}\right) & |t|\ll\tau_{imp}\\ y_d\left(\frac{|t|\tau_{imp}}{\tau_\lambda^2}\right) & |t|\gg\tau_{imp}\end{cases} \quad (10)$$

where

$$\tau_\lambda=\left(\frac{2k^2T}{M}\right)^{-1/2}. \quad (11)$$

For $d=3$, $y_3(x)=(1-e^{-x})/x$ [16]. For $d=2$

$$f_t=\int\frac{d\mathbf{s}'}{2\pi}f(k(s-s'),t). \quad (12)$$

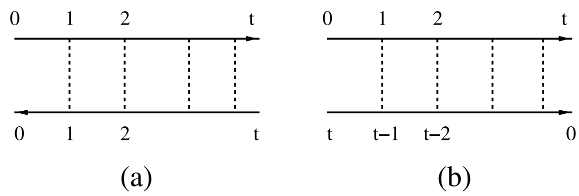

Fig. 1. Diagrams for the Diffuson (a) and the Cooperon (b). Solid line is dressed quantum particle propagator, dashed line connecting points $\mathbf{r},t$ and $\mathbf{r}',t'$ corresponds to $K(\mathbf{r}-\mathbf{r}',t-t')$.

Using the integral

$$\frac{1}{\pi}\int_0^\pi d\theta e^{-A(1-\cos\theta)}=e^{-A}I_0(A), \quad (13)$$

where I_0 is the modified Bessel function, we obtain

$$y_2(x)=e^{-x/2}I_0(x/2). \quad (14)$$

For $d=1$

$$y_1(x)=e^{-x/2}. \quad (15)$$

Equation (8) can be easily understood if we compare diagrams for the Diffuson (the sum of all ladder diagrams) and the Cooperon in Figure 1. The Diffuson does not have any mass because of the Ward identity. In the case of the Cooperon, the Ward identity is broken, and the difference $[1-f(t)]$ shows how strongly. The interaction line which dresses the single particle propagator is given by the static correlator, and the interaction line which connects two different propagators in a ladder is given by the dynamic correlator. The time-reversal invariance in the system we are considering is broken due to dephasing; the diffusion pole of the particle-particle propagator disappears, although the particle-hole propagator still has a diffusion pole, which is guaranteed by particle number conservation. (The weak localization effects for the case of inelastic electron-phonon scattering were discussed by Afonin et al. [18].)

Considering the limiting cases, from equation (8) we obtain:

(i) in the case $2k^2T/M\ll\tau/\tau_{imp}^3$

$$C_E(\mathbf{q})=\int_0^\infty \exp\left[-D(E)\mathbf{q}^2t-t^2/\tau_\varphi^2(E)\right]dt, \quad (16)$$

where

$$\tau_\varphi=\left(\frac{2M}{k^2T}\frac{\tau}{\tau_{imp}}\right)^{1/2}; \quad (17)$$

(ii) in the case $\tau/\tau_{imp}^3\ll 2k^2T/M\ll 1/\tau^2$

$$C_E(\mathbf{q})=\int_0^\infty \exp\left[-D(E)\mathbf{q}^2t-t^3/\tau_\varphi^3(E)\right]dt, \quad (18)$$

where

$$\tau_\varphi=\left(\frac{3M\tau}{k^2T}\right)^{1/3}. \quad (19)$$

Thus we obtain the crucial parameter – the dephasing time τ_φ.

The results for the dephasing time (up to a numerical factors of order one) can be understood using simple qualitative arguments. Consider the ballistic regime. If a single collision leads to the quantum particle energy change δE, the dephasing time could be obtained using equation (19)

$$\tau_\varphi \delta E \sqrt{\frac{\tau_\varphi}{\tau}} \sim 2\pi, \tag{20}$$

where τ_φ/τ is just the number of scatterings during the time τ_φ. So in this case

$$\frac{1}{\tau_\varphi^3} \sim \frac{(\delta E)^2}{\tau}. \tag{21}$$

If we notice that $1/\tau_\lambda$ is the averaged quantum particle energy change in a single scattering, δE, we immediately regain equation (19). Equations (20) and (21) also imply that if the scattering is quasi elastic (and slow motion of scatterers means just that), the energy relaxation time is much larger than the dephasing time [19]. Hence we have the right to ignore the Doppler caused cumulative energy shift, which otherwise would have lead to the appearance of the Diffuson mass.

Inserting equation (18) into the self-consistent equation, for the diffusion coefficient D we obtain

$$\frac{D_0(E)}{D(E)} = 1 + \frac{1}{4\pi^2 mk} \sum_{\mathbf{q}} C_E(\mathbf{q}) \tag{22}$$

where D_0 is the diffusion coefficient calculated in the Born approximation

$$D_0 = \frac{1}{d} v^2 \tau; \tag{23}$$

v is the particles velocity, and the momentum cut-off $|\mathbf{q}| < 1/\ell$ is implied, where $l = k\tau/m$ is the mean free path. Thus we obtain

$$\frac{D_0}{D} = 1 + \frac{1}{\pi mk} \int_0^\infty dt \int_0^{1/l} dq\, q^{d-1} \times \exp\left[-Dq^2 t - g(t/\tau_\varphi)\right], \tag{24}$$

where $g(x)$ is some function which goes to infinity when x goes to infinity as some power of x higher than one (in the particular case of ballistic regime $g(x) = x^3$, and in the diffusive regime $g(x) = x^2$).

Introducing dimensionless variables we obtain

$$\frac{D_0}{D} = 1 + \frac{1}{\pi} \frac{1}{(kl)^{d-1}} \int_0^\infty d\tilde{t} \int_0^1 d\tilde{q}\, \tilde{q}^{d-1} \times \exp\left[-\frac{1}{d}\frac{D}{D_0}\tilde{q}^2\tilde{t} - g(\tilde{t}\tau/\tau_\varphi)\right]. \tag{25}$$

Thus we have obtained an algebraic equation for D/D_0, which (equation) depends upon two parameters: $\tau_\varphi/\tau \gg 1$ and kl, which can be arbitrary.

Let us start analysis of this equation with the case $d = 2$. Calculating the integral with respect to $\tilde{q}$ we obtain

$$\frac{D}{D_0} = 1 - \frac{1}{\pi kl} \int_0^\infty \frac{d\tilde{t}}{\tilde{t}} \left[1 - e^{-\frac{D\tilde{t}}{2D_0}}\right] e^{-g(\tilde{t}\tau/\tau_\varphi)}. \tag{26}$$

Let us make the assumption (which we'll justify a posteriori)

$$D\tau_\varphi/D_0\tau \gg 1. \tag{27}$$

To calculate the integral

$$I(\lambda) = \int_0^\infty \frac{d\tilde{t}}{\tilde{t}} \left[1 - e^{-\lambda\tilde{t}}\right] e^{-g(\tilde{t})}, \qquad \lambda \gg 1, \tag{28}$$

let us divide the region of integration by choosing some x satisfying $1/\lambda \ll x \ll 1$. We obtain

$$\begin{aligned} I(\lambda) &= \left[\int_0^x + \int_x^\infty\right] \frac{d\tilde{t}}{\tilde{t}} \left[1 - e^{-\lambda\tilde{t}}\right] e^{-g(\tilde{t})} \\ &= \int_0^x \frac{d\tilde{t}}{\tilde{t}} \left[1 - e^{-\lambda\tilde{t}}\right] + \int_x^\infty \frac{d\tilde{t}}{\tilde{t}} e^{-g(\tilde{t})} \\ &= \ln(\lambda x) - \ln x = \ln\lambda. \end{aligned} \tag{29}$$

(In Eq. (29) we have ignored all numerical factors of order 1 in the argument of the logarithms.) Hence, equation (26) can be presented in the form

$$\frac{D}{D_0} = 1 - \frac{1}{\pi kl} \ln\left(\frac{D\tau_\varphi}{D_0\tau}\right). \tag{30}$$

Solution of equation (30) is particularly simple in two limiting cases: $l_\varphi \ll \xi$ and $l_\varphi \gg \xi$, where $l_\varphi = v\tau_\varphi$ is the dephasing length, and $\xi = le^{\frac{\pi kl}{2}}$ is the localization length [14]. In the former case we obtain just weak localization corrections

$$\frac{D}{D_0} = 1 - \frac{1}{\pi kl} \ln\frac{\tau_\varphi}{\tau}, \tag{31}$$

and in the latter case

$$D = \frac{\xi^2}{\tau_\varphi}. \tag{32}$$

We see that in both cases the assumption (27) is satisfied.

Results of a numerical solution of equation (26) for $g(x) = x^3$, $g(x) = x^2$ and $g(x) = x$ are presented in Figure 2. One can see that the curves for $D/D_)$ are practically indistinguishable. Thus the exact form of the function $g(x)$ is not important. All the relevant information is contained in the dephasing time, determined by the parameter τ_φ.

Notice, that the quantum diffusion of particles scattered by the slow moving scatterers turns out to be similar to the case when there are two separate scattering mechanisms: strong elastic scattering causing relaxation of momentum, and weak inelastic scattering due to say, phonons, causing dephasing (except for the definition of τ_φ). Strong dependence of the diffusion coefficient for

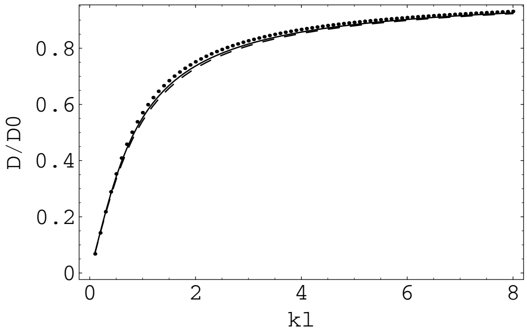

Fig. 2. The results of numerical solution of equation (26) ($\tau_\varphi/\tau = 10$) for $g(x) = x^2$ (solid line), $g(x) = x^3$ (dashed line), and $g(x) = x$ (dots).

$d = 2$ upon the ratio of the dephasing and the localization length (for the case of two scattering mechanisms) was thoroughly discussed in references [6,20–22].

As it was noticed by Gogolin and Zimanyi [20], there is a lower bound of temperature for the validity of equation (32). At low enough temperatures variable range hopping, which is of course not taken into account by the self-consistent localization theory is the main diffusion mechanism. So equation (24) is valid, provided

$$T \gg \Delta E, \tag{33}$$

where ΔE is the average energy difference between neighboring localized states. Equation (33) can be presented as

$$T \gg \frac{1}{ml^2} e^{-\pi kl}. \tag{34}$$

On the other hand, inequality $l_\varphi \gg \xi$ after substitution of equation (17) gives

$$T \ll \frac{M}{m} \frac{\tau}{\tau_{imp}} \frac{1}{ml^2} e^{-\pi kl}, \tag{35}$$

and after substitution of equation (19) gives

$$T \ll \frac{M}{m} \frac{1}{ml^2} e^{-\pi kl}. \tag{36}$$

We again see the importance of the large parameter M/m.

In fact, equation (32) is valid both for $d = 1$ and $d = 3$ (in the latter case, provided we have localization in the absence of dephasing). Taking into account the numerical results obtained for $d = 2$, for the purpose of semi-quantitative analysis we may approximate equation (24) by

$$\frac{D_0}{D} = 1 + \frac{1}{\pi mk} \int_0^\infty dt \int_0^{1/l} dq\, q^{d-1} \times \exp\left[-Dq^2 t - t/\tau_\varphi\right]. \tag{37}$$

Calculating the integral with respect to t we obtain equation (37) in the form

$$\frac{D}{D_0} = 1 - \frac{d}{\pi (kl)^{d-1}} \int_0^1 \frac{d\tilde{q}\tilde{q}^{d-1}}{\tilde{q}^2 + \frac{l^2}{D\tau_\varphi}}. \tag{38}$$

For $d = 2$ we obtain

$$\frac{D}{D_0} = 1 - \frac{1}{\pi kl} \ln\left[\frac{D\tau_\varphi}{l^2} + 1\right], \tag{39}$$

which in our approximation coincides with equation (30).

For $d = 1$ we obtain from equation (37)

$$\frac{D}{D_0} = 1 - \frac{1}{\pi} \frac{\sqrt{D\tau_\varphi}}{l} \tan^{-1} \frac{\sqrt{D\tau_\varphi}}{l}. \tag{40}$$

Again ignoring numerical multipliers of order 1 we obtain

$$D = D_0 \frac{\tau}{\tau_\varphi}. \tag{41}$$

If we take into account that for $d = 1$ we have $\xi \sim l$, we see that equation (41) is equivalent to equation (32). One must admit, however, that for $d = 1$ the self-consistent localization theory should be handled with care. In addition interaction between quantum particles, not considered in the present paper, may strongly influence the localization processes [23].

For $d = 3$ from equation (37) we obtain

$$\frac{D}{D_0} = 1 - \frac{3}{\pi (kl)^2} \left[1 - \frac{l}{\sqrt{D\tau_\varphi}} \tan^{-1} \frac{\sqrt{D\tau_\varphi}}{l}\right]. \tag{42}$$

One can see, that for $d = 3$ (similar to the case $d = 2$) equation (32) ceases to be valid when the localization length ξ becomes large enough, which happens when the parameter kl approaches the critical value $\sqrt{3/\pi}$ from below. In fact, in this region equation (42) can be presented as

$$\frac{D}{D_0} = 2\sqrt{3\pi}(\lambda_c - \lambda) + \frac{l}{\sqrt{D\tau_\varphi}} \tan^{-1} \frac{\sqrt{D\tau_\varphi}}{l}, \tag{43}$$

where $\lambda = 1/\pi kl$. and $\lambda_c = 1/\sqrt{3\pi}$. After assuming that the term $2\sqrt{3\pi}(\lambda_c - \lambda)$ can be ignored with respect to the second term in the rhs of equation (43), and that $D\tau_\varphi/l \gg 1$ we obtain

$$D = \frac{l^2}{\tau^{2/3}\tau_\varphi^{1/3}}. \tag{44}$$

Now checking the assumptions and taking into account that in the critical region [14] $\xi = l/|\lambda - \lambda_c|$, we see that equation (44) is valid, provided

$$\xi > l^{2/3} l_\varphi^{1/3}. \tag{45}$$

The results of numerical solution of equation (42) are presented in Figure 3.

Notice that in accordance with references [20,21] the dephasing time dependence of the diffusion coefficient can be obtained from its frequency dependence by replacing ω by $i\tau_\varphi$. Equation (24) in the absence of dephasing but for finite frequency is [14]

$$\frac{D_0}{D(\omega)} = 1 + \frac{1}{\pi mk} \int_0^\infty dt \int_0^{1/l} dq\, q^{d-1} \times \exp\left[-D(\omega)q^2 t + i\omega t\right]. \tag{46}$$

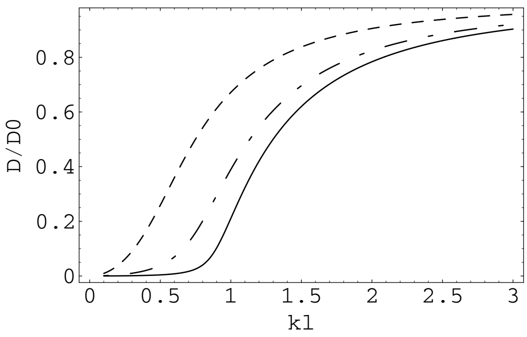

Fig. 3. The results of numerical solution of equation (42) for $\tau_\varphi/\tau = 10$ (dashed line), $\tau_\varphi/\tau = 100$ (dot-dashed line), and $\tau_\varphi/\tau = 1000$ (solid line).

The localization length is defined [14] as

$$\xi = \lim_{\omega\to 0}\sqrt{\frac{D(\omega)}{-i\omega}}. \tag{47}$$

Analyzing the solution qualitatively, we may substitute $1/\tau_\varphi$ for $-i\omega$ into the definition of the localization length (46) and obtain equation (32).

Conclusions

We considered the influence of slow random motion of random scatterers on the localization of quantum particles. It turned out that whenever the states of the quantum particles were localized, under the assumption, that the same scatterers are *static*, taking the motion of the scatterers into account leads to a finite value of the diffusion coefficient. In particular, for the three dimensional case, there exists a narrow crossover region in energy space, which separates the states with high and low diffusion coefficient, the latter being inversely proportional to the dephasing time (For the states with fast diffusion the dephasing is irrelevant). Like the position of the mobility edge in the case of static scatterers, the position of this crossover region is determined by the criterion that the mean free path is of the order of the quantum particle wavelength. This crossover region we call the quasi-mobility edge, and the phenomena in general we call quasi-localization. For the two dimensional case we have a transition between weak localization, observed when the dephasing length is less than the localization length (calculated for static scatterers), and strong localization observed in the opposite case.

The main application of our results we see as lying in the description of kinetics of ultracold gases. However, we would like to mention possible application of these results to at least one other field. In our previous publication [24], we studied the influence of dephasing on the Anderson localization of the electrons in magnetic semiconductors, driven by spin fluctuations of magnetic ions. There the role of heavy particles was played by magnons; complete spin polarization of conduction electrons prevented magnon emission or absorption processes, and only the processes of electron-magnon scattering being allowed.

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